

Monopoles in ferromagnetic metals

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Abstract

The aim of this short review is to give an introduction to monopoles and to present theoretical derivation of two particular monopoles in ferromagnetic metals, a hedgehog monopole and a spin damping monopole. In an electromagnetism in the vacuum, described by Maxwell's equations, magnetic field and electric field are not symmetric, since there is no monopole, a particle having a finite magnetic charge. Still, monopole has been an exciting object for a long time, and was discussed on a phenomenological ground by Dirac in 1931. A theoretical possibility of monopole generation was first given by 't Hooft and Polyakov in 1974 in a context of a symmetry breaking in a grand unified theory (GUT). A GUT monopole has not been discovered in experiments so far. In contrast to in the vacuum, several kinds of monopoles are expected to emerge in solids associated with various symmetry breaking mechanisms. Of particular interest is metallic ferromagnetic systems, since a breaking of a symmetry of conduction electron spin, described by an $SU(2)$ algebra, can give rise to monopoles. Indeed two monopoles are theoretically predicted in ferromagnets; one is a hedgehog monopole arising from a topological spin structure and the other arising from spin damping in the presence of spin-orbit interaction. In this paper, we focus on these monopoles, while other objects similar to monopoles but not coupled to effective electromagnetic fields, such as spin ice monopoles, are touched only briefly in the introduction. Those monopoles are extended objects coupled to effective electromagnetic fields, which are described by Maxwell's equations with monopole contribution. The effective fields are the one coupled to spin of a particle like electron, muon and neutron; two monopoles are thus detectable by electric measurements. Spin damping monopoles can be generated in simple systems such as a junction of a ferromagnet and a heavy element with strong spin-orbit interaction such as Pt. This monopole is essential in coupling electronics with magnetism, and is thus expected to play an essential role in spintronics.

Keywords: Monopole, Spintronics, Spin current, Maxwell's equations

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FIG. 1. Left: Schematic figure of a magnet and magnetic field around it. Right: A monopole, particle having only N (or S) magnetic pole, and the magnetic field it emits.

I. INTRODUCTION

A. Monopole

Macroscopic magnets have two poles called N (north) and S (south) at two different edges (Fig. 1). We also know that we cannot extract only N or S pole from magnets; when we divide a magnet, a pair of N and S poles is created at the edge and each piece becomes a magnet with the equal amount of N and S poles. There is therefore no monopole at least in the energy scale we concern in our life. This is because the magnets are made of spin, a quantum magnet the electron carries, which creates the divergencelss magnetic field only. In other words, each spin contain N and S poles, which are not separable since elementary particles are pointlike.

Possibility of a particle carrying only N or S pole, called magnetic monopole, was first discussed by Dirac [1]. He showed based on a macroscopic consideration that monopole is tied to a string of singularity, and its existence is allowed only if the charge of the monopole is quantized by unit of $2\pi\hbar/e$ (e is the electron charge). A microscopic mechanism of monopole creation was theoretically discovered by 't Hooft [2] and Polyakov [3] independently. They showed that monopole arises in a grand unified theory (GUT) of elementally particles when the symmetry breaking of the SU(5) symmetry to the U(1) symmetry of the electromagnetic field occurs. Such monopoles may have been created in the early universe (at about 10^{-10} sec after the big bang) and may still be around us. So far, however, no evidence has been obtained in experiments waiting for a monopole from universe to go through superconducting

detectors [4] or detecting the ionization [5]. The energy scale of the GUT monopole is 10^{17} GeV, and so creating one in accelerator on the earth is impossible.

B. Maxwell's equations without monopole

Before starting to discuss monopoles, let us discuss the law of conventional electromagnetism without monopole. We know that electric field, \mathbf{E} , and magnetic field, \mathbf{B} , behave differently, i.e., electric field has finite divergence while magnetic field is divergenceless [6]. To put in equations,

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon} \\ \nabla \cdot \mathbf{B} &= 0,\end{aligned}\tag{1}$$

where ρ is charge density and ϵ is dielectric constant. The two fields are governed by another set of equations describing rotational components;

$$\begin{aligned}\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu \mathbf{j} + \epsilon \mu \frac{\partial \mathbf{E}}{\partial t},\end{aligned}\tag{2}$$

where μ is magnetic permeability and \mathbf{j} is current density. The fields \mathbf{E} and \mathbf{B} are therefore not symmetric; magnetic field is generated by electric current by the Ampère's law, while there is no driving current for electric field. One may imagine that the electromagnetism law is more beautiful if there is a current that drives electric field via an analog of the Ampère's law. Let us write such current as \mathbf{j}_m . The first equation of Eq. (2) then becomes

$$\nabla \times \mathbf{E} = -\mathbf{j}_m - \frac{\partial \mathbf{B}}{\partial t}.\tag{3}$$

Taking the divergence of this equation, we have

$$0 = -\nabla \cdot \mathbf{j}_m - \frac{\partial}{\partial t}(\nabla \cdot \mathbf{B}),\tag{4}$$

which indicates that the second equation of Eq. (1) needs to be modified as

$$\nabla \cdot \mathbf{B} = \rho_m,\tag{5}$$

where ρ_m is a quantity satisfying a conservation law of

$$\dot{\rho}_m + \nabla \cdot \mathbf{j}_m = 0.\tag{6}$$

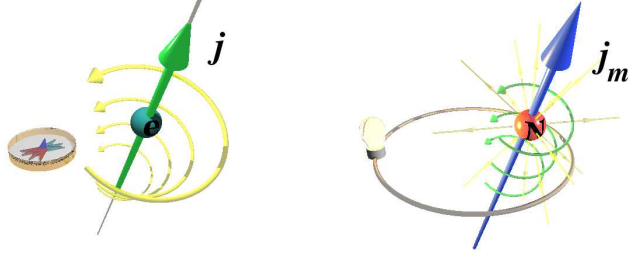


FIG. 2. Left) The Ampère's law allows us to generate magnetic field from the electric current, \mathbf{j} . In contrast, there is no Ampère's law for the electric field in the electromagnetism if U(1) symmetry is exact. The exception is the case with monopole: monopole current, \mathbf{j}_m , generates \mathbf{E} (Right).

Equation (5) indicates that ρ_m emits a magnetic field like a N or S pole; hence ρ_m is a density of monopole. The current \mathbf{j}_m is then a monopole current. Thus, if monopole exists, \mathbf{E} and \mathbf{B} become symmetric, and the electric field can be generated by applying the monopole current.

The aim of this short review is to give an introductory description of monopoles in condensed matter and explain in detail monopoles in ferromagnetic metals. The paper is organized as follows. Monopole is discussed from a macroscopic viewpoint in §II. Section III is a brief introduction of monopoles in solids. Other topological objects similar to monopoles are introduced in §IV. These objects are touched only briefly, since we would like in this paper to focus on monopoles which really couple to electromagnetism via Maxwell's equations. Sections V and VI are the main content of the paper presenting theoretical description of hedgehog monopole and spin damping monopole, respectively. Relativistic notation is summarized in §A and spin damping is briefly described in §B.

II. MONOPOLE FROM MACROSCOPIC VIEWPOINT : DIRAC'S STRING

Let us go on to discuss monopoles from a macroscopic viewpoint, i.e., based on the Maxwell's equations. As is well-known, in the electromagnetism without monopole, the magnetic field \mathbf{B} is expressed as a rotation of a vector potential, namely, as $\mathbf{B} = \nabla \times \mathbf{A}$. When monopole is present, this is no longer true, but still one can define a vector potential

as a line integral over a certain line L starting from monopole to the infinity. Let us consider a case of a single monopole at $\mathbf{r} = 0$,

$$\mathbf{B}(\mathbf{r}) = \frac{g}{4\pi} \frac{\mathbf{r}}{r^3}, \quad (7)$$

where g is a monopole charge. We can define a "vector potential"

$$A_i^{(L)}(\mathbf{r}) = \sum_{jk} \epsilon_{ijk} \int_L dr'_j B_k(\mathbf{r} - \mathbf{r}'), \quad (8)$$

where B_k is the k th component of the magnetic field (including monopole) and L is a string connecting $\mathbf{r} = 0$ and the infinity. The "magnetic field" of this vector potential $\mathbf{A}^{(L)}$ then reads

$$\nabla \times \mathbf{A}^{(L)}(\mathbf{r}) = \mathbf{B}(\mathbf{r}) + \int_L d\mathbf{r}' \rho_m(\mathbf{r} - \mathbf{r}'), \quad (9)$$

where ρ_m is the monopole density, $\rho_m(\mathbf{r}) = g\delta^3(\mathbf{r})$ if a single monopole case. The first term of Eq. (9) correctly reproduces the field \mathbf{B} but we have an additional non local singular field represented by the second term. The singular magnetic field represented by the second term of Eq. (9) exists along the string L . Dirac argued therefore that monopole needs to have such string (called the Dirac's string) but this string must not be "seen" by physical particles [1]. If many monopoles exist, many strings L each attached to a single monopole are needed to describe the vector potential.

The unobservability of the string is ensured if the monopole charge g is quantized to certain values. One way to derive the quantization condition is to impose the condition that the shift of the Dirac's string does not modify the wave function of the charged particles [6]. Equation (8), explicitly written in a single monopole case reads

$$A_i^{(L)}(\mathbf{r}) = \frac{g}{4\pi} \sum_{jk} \epsilon_{ijk} \int_L dr'_j \frac{(r - r')_k}{|\mathbf{r} - \mathbf{r}'|^3}. \quad (10)$$

If one shift the string L to be another string L' , the vector potential changes to

$$A_i^{(L')}(\mathbf{r}) = A_i^{(L)}(\mathbf{r}) + \frac{g}{4\pi} \sum_{jk} \epsilon_{ijk} \int_C dr'_j \frac{(r - r')_k}{|\mathbf{r} - \mathbf{r}'|^3}, \quad (11)$$

where $C = L' - L$ is a closed path surrounded by L and L' . The last term of the right-hand side is proportional to the derivative of the solid angle $\Omega_C(\mathbf{r})$ subtended by C observed at \mathbf{r} .

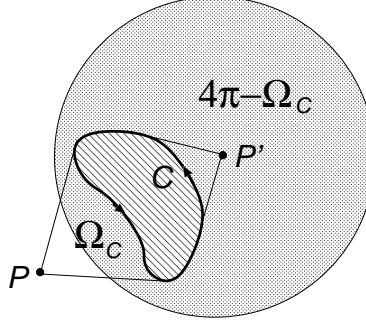


FIG. 3. When a solid angle of a closed path C observed at P is Ω_C , it is $4\pi - \Omega_C$ at P' in the opposite side of the path.

In fact, a difference of solid angles observed at \mathbf{r} and $\mathbf{r} + \delta\mathbf{r}$, which we call $\delta\Omega_C$, is (noting that $d\mathbf{r}' \times \delta\mathbf{r}$ is a vector normal to the plane having a length of an element of area)

$$\delta\Omega_C = \int_C (d\mathbf{r}' \times \delta\mathbf{r}) \cdot \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r} - \mathbf{r}'|^3}. \quad (12)$$

We therefore see that

$$\nabla\Omega_C = \int_C d\mathbf{r}' \times \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}, \quad (13)$$

and that Eq. (11) becomes

$$\mathbf{A}^{(L')} = \mathbf{A}^{(L)} + \frac{g}{4\pi} \nabla\Omega_C. \quad (14)$$

Since a phase for the electron's wave function is given by an integral of the vector potential, this modification of the vector potential results in a phase change of $e^{\frac{ie}{\hbar} \frac{g}{4\pi} \Omega_C}$. Quantum mechanics requires that this phase is single-valued. However, solid angle Ω_C is not single-valued. Multi-valuedness of Ω_C occurs when Ω_C changes by 4π as an electron goes through the plane spanned by C (Fig. 3). The single-valuedness of the electron's phase is thus ensured by requiring that

$$eg = 2\pi n\hbar, \quad (15)$$

where n is an integer. This is the quantization condition for the monopole charge, pointed out by Dirac.

The above argument is based on the assumption that the magnetic field is written as a rotation of a $U(1)$ vector potential even in the presence of monopoles. This assumption,

however, is wrong, since the exact U(1) gauge invariance does not exist when monopole exists. As we will show later, the monopole field in the case of the symmetry breaking mechanism is expressed by the additional field orthogonal to the U(1) degrees of freedom. The Dirac's string containing a singular magnetic field (the last term of Eq. (9)), is thus an artifact arising when one tries to describe the monopole field within the U(1) gauge theory assuming $\mathbf{B} = \nabla \times \mathbf{A}$, and it is eliminated when one takes account of microscopic origin of the monopole.

In the case of 't Hooft-Polyakov monopole, monopole is created from the symmetry breaking [7]. Their argument is for a system of non-Abelian gauge field coupled to a Higgs represented by a vector field, ϕ^α ($\alpha = x, y, z$). They showed that there is a solution where Higgs field behaves at the infinity as

$$\phi^\alpha \propto \frac{r^\alpha}{r} \quad (r \rightarrow \infty). \quad (16)$$

In this configuration, magnetic field is shown to contain monopole, and monopole density is determined by Higgs field as

$$\rho_m \propto \sum_{ijk} \sum_{\alpha\beta\gamma} \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} \partial_i \phi^\alpha \partial_j \phi^\beta \partial_k \phi^\gamma. \quad (17)$$

A volume integral of this monopole density is written as a surface integral at the infinity, and it turns out to take only integer value called a winding number. This topological nature of the Higgs field leads to a quantization of monopole charge. Thus, 't Hooft-Polyakov monopole emerges from a symmetry breaking from non-Abelian group to U(1) group driven by a condensation of Higgs field. Dirac's string is not necessary here, since monopole field is created from Higgs field, which is different from a gauge field.

In §V, we will discuss in detail a monopole arising from a symmetry breaking in ferromagnets.

III. MONOPOLES IN SOLIDS

A. Hedgehog monopole

As we have mentioned, in the electromagnetism in the vacuum, monopole predicted at high energy has not been found. Even if it is found in the future, we cannot make a device

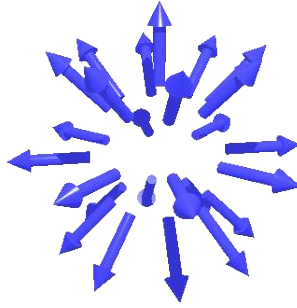


FIG. 4. Schematic illustration of a magnetization structure for a hedgehog monopole, discussed by Volovik. This structure has a singularity at the center, and is topologically non-trivial. The electron coupled to this magnetization structure feels the effective electromagnetic field with a monopole.

since its energy is too high. In contrast to electromagnetism in the vacuum, we have hope in solids. In fact, electrons in solids feel another $U(1)$ gauge field which couples to the electron's spin [8]. The spins are object in an $SU(2)$ space, and thus if the breaking of its symmetry occurs, a resulting effective $U(1)$ gauge field may contain monopoles according to the 't Hooft and Polyakov scenario. In addition, the energy scale of the symmetry breaking is at low energy, less than 1eV, and so device application would be straightforward.

Such a monopole was indeed theoretically pointed out to exist in ferromagnetic metals by Volovik in 1987 [9]. In fact, a monopole arises from a strong sd coupling between the conduction electron and local spin, which specifies the projection of the conduction electron spin having $SU(2)$ symmetry to a $U(1)$ space. When local spin structure is a topologically non-trivial one called a hedgehog, the projected effective electromagnetic field contains monopoles.

B. Spin damping monopole

Recently, another monopole in ferromagnetic metals was predicted [10, 11]. The idea is to include spin-orbit interaction besides sd interaction. Spin-orbit interaction modifies the projection to the $U(1)$ plane defined by the sd interaction, and thus new monopole may arise

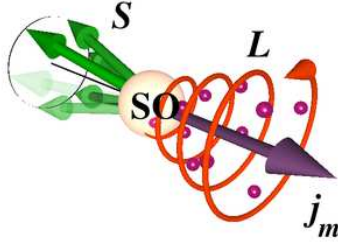


FIG. 5. Schematic illustration of a spin damping monopole. A damping of spin (\mathbf{S}) generates an orbital angular momentum \mathbf{L} for electrons when spin-orbit interaction (SO) acts. A circular motion due to the orbital angular momentum is equivalent to that caused by an electric field induced by a monopole current \mathbf{j}_m via Ampère's law.

when the spin-orbit interaction is included. The monopole generation in such case cannot be discussed by use of gauge fields, since the spin-orbit interaction is not a gauge interaction. Instead, novel method based on a transport calculation was applied by Takeuchi and Tatara [10]. They derived effective electromagnetic fields by calculating the electric current by use of Keldysh Green's functions, and showed that the fields satisfy Maxwell's equation with monopole. In their unique approach, a knowledge of gauge invariance was not necessary to explore the structure of the electromagnetism. This fact may sound surprising, but is natural, since a U(1) gauge invariance is equivalent to charge conservation law, which is strictly observed in transport calculations. In their case, a deviation from exact U(1) invariance due to spin-orbit interaction resulted in a monopole contribution. Based on the transport method, they revealed that monopole arises when the spin structure is dynamic and when spin-orbit interaction is included. More specifically, monopole arises when there is a damping of spin, represented by a damping vector (see §B),

$$\mathbf{N} \equiv \mathbf{S} \times \dot{\mathbf{S}}, \quad (18)$$

(\mathbf{S} being local spin), and thus the monopole was named the spin damping monopole.

A physical mechanism of spin damping monopole is understood as a transfer of the spin angular momentum to orbital one (Fig. 5). In fact, a vector $\alpha\mathbf{N}$, where α is the Gilbert damping parameter (proportional to the strength of the spin-orbit interaction if in metals), represents the spin angular momentum dissipated. This lost angular momentum is converted into the orbital motion of the electrons by the spin-orbit interaction, inducing the circular

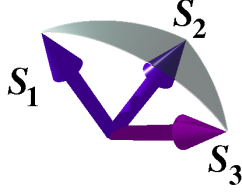


FIG. 6. A scalar spin chirality subtended by three spins represents non-coplanarity. Spin chirality reduces to spin Berry's phase in the slowly varying limit.

orbital motion of the electrons. This circular motion is regarded as a result of a fictitious magnetic field due to a monopole. From this intuitive explanation, the monopole density should appear when the spin damping is spatially inhomogeneous. This is indeed consistent with the result of Ref. [10], which showed that the monopole density is $\rho_m \propto \nabla \cdot \mathbf{N}$. Similarly, when the spin damping is time-dependent, a monopole current arises since a temporal change of effective magnetic field is equivalent to a monopole current. The spin damping monopole was argued to be essential in the spin-charge conversion in spintronics phenomena such as the inverse spin Hall effect [12].

These monopoles in ferromagnetic systems are discussed in detail in §V and §VI.

IV. RELATED TOPOLOGICAL OBJECTS IN SOLIDS

A. Spin chirality

There are several objects closely related to monopoles. Spin chirality is one example. In a spin system on a lattice, a non-coplanarity of three spins, \mathbf{S}_1 , \mathbf{S}_2 and \mathbf{S}_3 , is represented by a scalar product called a scalar chirality, $\chi \equiv \mathbf{S}_1 \cdot (\mathbf{S}_2 \times \mathbf{S}_3)$ (Fig. 6). When the spin structure is slowly varying in space, spin chirality can be expanded as

$$\mathbf{S}(\mathbf{x}) \cdot (\mathbf{S}(\mathbf{x}_1) \times \mathbf{S}(\mathbf{x}_2)) \simeq \sum_{\mu\nu} (x_1 - x)_\mu (x_2 - x)_\nu \mathbf{S}(\mathbf{x}) \cdot (\partial_\mu \mathbf{S}(\mathbf{x}) \times \partial_\nu \mathbf{S}(\mathbf{x})), \quad (19)$$

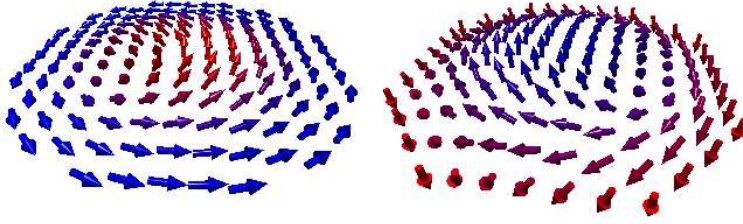


FIG. 7. Schematic illustration of a magnetic vortex (left) and a skyrmion (right). These are two-dimensional structures of magnetization with topological meaning. The topological number of a vortex is the number of spin rotation as one travels along the perimeter, called a vorticity. In a skyrmion structure, the spins at the perimeter is fixed to be parallel (spin down in this figure), and its topological number is the number of spin rotation as one travels from the origin to the infinity along the radial direction. Vortex and skyrmion in this figure have a topological number of one.

and the spin chirality reduces to the spin Berry phase. In the continuum limit, therefore, a scalar chirality is represented by a vector

$$\chi_i \equiv \sum_{jk} \epsilon_{ijk} \mathbf{S} \cdot (\nabla_j \mathbf{S} \times \nabla_k \mathbf{S}), \quad (20)$$

where the direction i is orthogonal to a plane the three spins lie. This expression of the scalar chirality is proportional to spin Berry's phase or a local effective magnetic field generated by a hedgehog monopole (see Eq. (46)). One should note, however, that finite local spin chirality does not necessarily mean that a monopole exists; monopole existence requires a surface integral of the chirality, $\int d\mathbf{S} \cdot \boldsymbol{\chi}$ ($d\mathbf{S}$ is an element of surface integral), to be finite, and this is realized only for a three-dimensional hedgehog structure shown in Fig. 4.

Even when monopoles do not exist, a finite spin chirality leads to interesting physics, such as inducing chirality-driven exotic anomalous Hall effect [13, 14]. When spin structure such as domain wall is dynamic, the effective electric field acting on the electron's spin, the spin motive force, is induced (see Eq. (46)). Spin motive force was detected in the case of moving domain wall [15].

A monopole was also found in systems of conventional anomalous Hall effect driven by spin-orbit interaction [16, 17]. This monopole is a singularity in the momentum space and is not coupled to the electromagnetism by Maxwell's equations.

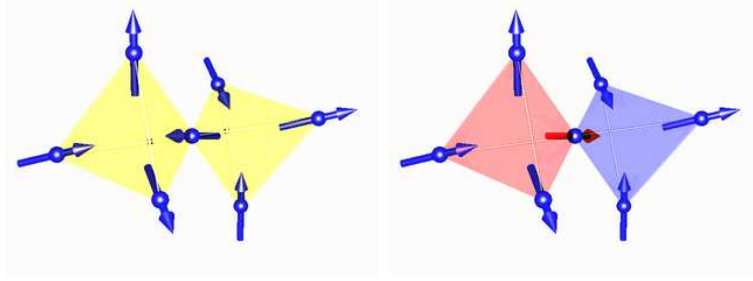


FIG. 8. Left: Ground state spin configuration in a frustrated magnet with a pyrochlore lattice. All the tetrahedra contains two in-spins and two out-spins, satisfying ice rule. Right: An excited state of a spin ice. Due to a flip of a spin connecting the two tetrahedra, left tetrahedra has three out-spins and the right tetrahedra has only one out-spin. This configuration contains effective monopoles; positive and negative ones on the left and right tetrahedra, respectively.

B. Vortex and skyrmion

Magnetic vortex and skyrmion in magnets are spin structures having topological feature in two-dimensions (Fig. 7) [18]. They have finite value of local spin chirality. A vortex is a structure where the number of spin rotation as one travels along the circle at the infinity (or the edge of the disk) is the topological number called a vorticity, which is an integer. Vortex state is stable in a small circular disk of submicron size since the magnetostatic energy at the edge is the lowest for the vortex state [19]. Switching of a vortex core by an electric current is a hot issue from a viewpoint of fundamental science and application to non-volatile memories. At present, rather large current density of $3.5 \times 10^{11} \text{ A/m}^2$ is necessary for a core flip, and the switching time is not very fast, about 20ns [20].

Skyrmion was originally proposed in high energy physics. In condensed matter, a magnetization structure shown in Fig. 7 is called a skyrmion [21]. Such a structure was predicted to arise when the inversion symmetry is broken. Skyrmions forming a lattice were observed in MnSi [22, 23]. Vortices and skyrmions have finite effective magnetic field, \mathbf{B}_s , but these topological structures are nothing to do with monopole (ρ_m and \mathbf{j}_m vanish), since they are two-dimensional objects.

C. Spin ice monopole

Monopoles in frustrated magnets with pyrochlore lattice is a hot subject recently [24]. In the ground state of pyrochlore spin system, each tetrahedra has two spins pointing to the center of tetrahedra (in-spin) and two spins pointing outwards (out-spin), satisfying so called an ice rule. When the system is excited, ice rule is broken and some of tetrahedra contain three in-spins and there arise the same number of tetrahedra with three out-spins (Fig. 8). In such excited states, tetrahedra with broken ice rule have nonvanishing divergence of magnetization, $\nabla \cdot \mathbf{M} \equiv \rho_M$. In terms of magnetic field $\mathbf{H} \equiv \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$, the excited states of spin ice systems thus have monopoles, i.e., $\nabla \cdot \mathbf{H} = -\rho_M$ [25]. The spin configuration corresponding to monopoles has been observed by neutron scattering experiment [26]. In addition, an effective magnetic charge of a spin ice monopole was measured in $\text{Dy}_2\text{Ti}_2\text{O}_7$ [27, 28]. Spin ice system is also created recently on an artificial square lattice [29].

One should understand that spin ice monopole is not a real monopole, since $\nabla \cdot \mathbf{B} = 0$ is strictly true even in the presence of any spin configurations; it is a monopole of an artificial magnetic field [24]. Nevertheless, an spin ice monopole interpretation is highly useful to describe excited states in frustrated spin systems from the viewpoint of spin liquid.

D. Monopole in topological insulator

Recent study revealed that monopole arises at the surface of topological insulators as a result of a image magnetic charge when an electric charge is close to a surface according to the following mechanism [30, 31]. A surface of topological insulator is described by a massless Dirac Hamiltonian in (2+1) dimensions. In this system, a parity anomaly arises from a ultra violet divergence, resulting in a Hall current perpendicular to the applied field

$$j^\mu = \frac{e^2 m}{2h|m|} \sum_{\mu\nu\rho} \epsilon^{\mu\nu\rho} \partial_\nu A_\rho, \quad (21)$$

where \mathbf{A} represents an vector potential and m is a topological mass [32]. An electric charge emitting a radial electric field outside a topological insulator thus creates a circulating electric current on the surface. A magnetic field generated by this current is equivalent to the one emitted by a magnetic charge inside topological insulator. Therefore, electric charge in a proximity with a topological insulator is coupled with a mirror magnetic charge (monopole),

forming a state called a dyon. This coupling of electric and magnetic charges is explained also by an effective Lagrangian of a topological insulator in three space dimensions (called a θ term) [30]

$$L_\theta = \frac{\theta}{2\pi} \int d^3x \mathbf{E} \cdot \mathbf{B}, \quad (22)$$

where θ is a constant. This Lagrangian, derived by integrating out the electrons, indicates that electric field and magnetic field are coupled directly, i.e., there is a magneto-electric effect. Lagrangian L_θ is written as a surface integral, and reduces to a Lagrangian which describes a parity anomaly in (2+1) dimensions.

V. HEDGEHOG MONOPOLE IN FERROMAGNETS

A. Gauge field representation of ferromagnetic metals

In this section, we derive a hedgehog monopole in a metallic ferromagnet in a strongly spin-polarized case. There are conduction electrons and local spins (magnetization). Local spin we consider is a classical vector field, represented by $\mathbf{S}(\mathbf{r}, t)$, which depends on space coordinate, \mathbf{r} , and time, t . We use a field (second-quantized) representation, where $c \equiv (c_+, c_-)^t$ (lower index \pm denotes spin and t stands for transpose) and c^\dagger represent the annihilation and creation operators for a conduction electron, respectively. The free part of conduction electron Hamiltonian is

$$H_0 = \int d^3r c^\dagger \left(-\frac{\hbar^2}{2m} \nabla^2 - \epsilon_F \right) c, \quad (23)$$

where m is the electron mass and ϵ_F is the Fermi energy. In metallic ferromagnets, conduction electrons are spin-polarized by local spin, \mathbf{S} , due to a coupling of sd -type given by

$$H_{sd} = -J \int d^3r \mathbf{S} \cdot (c^\dagger \boldsymbol{\sigma} c), \quad (24)$$

where J is a coupling constant, and $(c^\dagger \boldsymbol{\sigma} c)$ is the spin density of the electron ($\boldsymbol{\sigma} = \sigma_x, \sigma_y, \sigma_z$ represents Pauli matrix). The total Lagrangian of conduction electrons, defined as $L \equiv \int d^3r i\hbar c^\dagger \dot{c} - H$, where $H = H_0 + H_{sd}$, is

$$L = \int d^3r \left[i\hbar c^\dagger \dot{c} - \left(\frac{\hbar^2}{2m} |\nabla c|^2 - \epsilon_F c^\dagger c \right) + M \mathbf{n} \cdot (c^\dagger \boldsymbol{\sigma} c) \right], \quad (25)$$

where $M \equiv JS$ and $\mathbf{n} \equiv \mathbf{S}/S$ is a unit vector. In ferromagnetic metals, spin polarization of conduction electron satisfies $M\tau/\hbar \gg 1$, where τ is elastic lifetime of conduction electron. This is the adiabatic condition for disordered metals[8, 33, 34]. In this limit, a local gauge transformation to choose the electron spin quantization axis along $\mathbf{S}(\mathbf{r}, t)$ at each point is useful [33]. The deviation from perfect adiabaticity is then described by an SU(2) gauge field, which is small and we treat it perturbatively. A new electron operator $a \equiv (a_+, a_-)^t$ is defined as

$$c(\mathbf{r}, t) \equiv U(\mathbf{r}, t)a(\mathbf{r}, t), \quad (26)$$

where U is a 2×2 matrix which we further define as

$$U(\mathbf{r}, t) \equiv \mathbf{m} \cdot \boldsymbol{\sigma}, \quad (27)$$

\mathbf{m} being a real three-component unit vector we will define later. The matrix satisfies $U^2 = 1$, i.e., $U(\mathbf{r}, t)^{-1} = U(\mathbf{r}, t)$. A derivative of an operator c reads

$$\partial_\mu c(\mathbf{r}, t) = U(\mathbf{r}, t)(\partial_\mu + U(\mathbf{r}, t)^{-1}\partial_\mu U(\mathbf{r}, t))a = U(\mathbf{r}, t)(\partial_\mu + iA_{s,\mu})a, \quad (28)$$

where a gauge field (represented by a 2×2 matrix) is defined as

$$A_{s,\mu} \equiv -iU(\mathbf{r}, t)^{-1}\partial_\mu U(\mathbf{r}, t). \quad (29)$$

In terms of spin components, A_μ is written as [33]

$$A_{s,\mu} = (\mathbf{m} \times \partial_\mu \mathbf{m}) \cdot \boldsymbol{\sigma} \equiv \sum_\alpha A_{s,\mu}^\alpha \sigma_\alpha. \quad (30)$$

By the above gauge transform, the electron spin is transformed to be

$$U^{-1}\boldsymbol{\sigma}U = 2\mathbf{m}(\mathbf{m} \cdot \boldsymbol{\sigma}) - \boldsymbol{\sigma}. \quad (31)$$

The aim of our gauge transform is to let this spin to be along z -axis, i.e., $U^{-1}(\mathbf{n} \cdot \boldsymbol{\sigma})U = \sigma_z$.

This is satisfied if we choose

$$\mathbf{m} = \left(\sin \frac{\theta}{2} \cos \phi, \sin \frac{\theta}{2} \sin \phi, \cos \frac{\theta}{2} \right), \quad (32)$$

where (θ, ϕ) are the polar coordinates of \mathbf{S} . The gauge field is then obtained in a matrix notation with respect to spin index as

$$\begin{pmatrix} A_{s,\mu}^x \\ A_{s,\mu}^y \\ A_{s,\mu}^z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\partial_\mu \theta \sin \phi - \sin \theta \cos \phi \partial_\mu \phi \\ \partial_\mu \theta \cos \phi - \sin \theta \sin \phi \partial_\mu \phi \\ (1 - \cos \theta) \partial_\mu \phi \end{pmatrix}. \quad (33)$$

The electron part of Lagrangian is written in terms of a -electron as

$$L = \int d^3r \left[i\hbar a^\dagger \dot{a} - \frac{\hbar^2}{2m} |\nabla a|^2 + \epsilon_F a^\dagger a - M a^\dagger \sigma_z a + i \frac{\hbar^2}{2m} (a^\dagger A_{s,i} \nabla_i a - (\nabla_i a^\dagger) A_{s,i} a) - \frac{\hbar^2}{2m} (\mathbf{A}_s)^2 a^\dagger a - \hbar a^\dagger A_{s,0} a \right]. \quad (34)$$

In the matrix notation of the spin,

$$L = \int d^3r \left[(a^\dagger_+, a^\dagger_-) \left(i\hbar \partial_t + \frac{\hbar^2}{2m} \nabla^2 + \epsilon_F - \frac{\hbar^2}{2m} (\mathbf{A}_s)^2 \right) \begin{pmatrix} a_+ \\ a_- \end{pmatrix} + (a^\dagger_+, a^\dagger_-) \begin{pmatrix} -M + i \frac{\hbar^2}{2m} A_{s,i}^z \overleftrightarrow{\nabla}_i - \hbar A_{s,0}^z & i \frac{\hbar^2}{2m} A_{s,i}^+ \overleftrightarrow{\nabla}_i - \hbar A_{s,0}^+ \\ i \frac{\hbar^2}{2m} A_{s,i}^- \overleftrightarrow{\nabla}_i - \hbar A_{s,0}^- & M - i \frac{\hbar^2}{2m} A_{s,i}^z \overleftrightarrow{\nabla}_i + \hbar A_{s,0}^z \end{pmatrix} \begin{pmatrix} a_+ \\ a_- \end{pmatrix} \right] \quad (35)$$

where

$$A_{s,\mu}^\pm \equiv A_\mu^x \pm i A_\mu^y, \quad (36)$$

and $\overleftrightarrow{\nabla} \equiv \overrightarrow{\nabla} - \overleftarrow{\nabla}$ acts only to the field operators.

The adiabatic limit is defined as $M \rightarrow \infty$. In this limit, the minority spin electron has infinitely high energy and thus does not exist. Off-diagonal elements of Eq. (35) are accordingly neglected, and the system reduces to an electron interacting with a U(1) gauge field, $A_{s,\mu}^z$, described by a Lagrangian ($(\mathbf{A}_s^x)^2$ and $(\mathbf{A}_s^y)^2$ act only as a potential)

$$L_{\text{ad}} = \int d^3r a^\dagger_+ \left[i\hbar \partial_t + \frac{\hbar^2}{2m} \nabla^2 + \epsilon_F - M + i \frac{\hbar^2}{2m} A_{s,i}^z \overleftrightarrow{\nabla}_i - \frac{\hbar^2}{2m} (\mathbf{A}_s)^2 - \hbar A_{s,0}^z \right] a_+. \quad (37)$$

B. Hedgehog monopole arising from non-adiabaticity

When M is finite, perpendicular fluctuation represented by $A_{s,\mu}^\pm$ exists, and there is a finite deviation from the U(1) symmetry. These components appears in the U(1) space as a singular magnetic structure, i.e., a monopole. This can be shown as follows. (The notation in this subsection is a relativistic one [7], and upper and lower indices have different meanings. See Sec. A for details.) A field strength of the SU(2) gauge fields is

$$F_s^{\mu\nu} \equiv \sum_{\alpha=x,y,z} F_s^{\mu\nu,\alpha} \sigma_\alpha, \quad (38)$$

where α -component is given as

$$F_s^{\mu\nu,\alpha} \equiv \partial^\mu A_s^{\nu,\alpha} - \partial^\nu A_s^{\mu,\alpha} + (\mathbf{A}_s^\mu \times \mathbf{A}_s^\nu)^\alpha = \partial^\mu A_s^{\nu,\alpha} - \partial^\nu A_s^{\mu,\alpha} + \sum_{\beta\gamma=x,y,z} \epsilon_{\alpha\beta\gamma} A_s^{\mu,\beta} A_s^{\nu,\gamma}, \quad (39)$$

where $\epsilon_{\alpha\beta\gamma}$ is the asymmetric tensor in three-dimensions. By definition, the field strength satisfies the following identity called the Bianchi identity:

$$\epsilon^{\mu\nu\rho\sigma} D_\nu F_{s,\rho\sigma} = 0, \quad (40)$$

where $\epsilon_{\mu\nu\rho\sigma}$ is the asymmetric tensor in four-dimensions and

$$(D_\nu F_{s,\rho\sigma})^\alpha \equiv \partial_\nu F_{s,\rho\sigma}^\alpha + \sum_{\beta\gamma} \epsilon_{\alpha\beta\gamma} A_{s,\nu}^\beta F_{s,\rho\sigma}^\gamma, \quad (41)$$

represents the covariant derivative of field strength.

When away from the perfect adiabatic limit, $A_{s,\nu}^x = A_{s,\nu}^y$ need to be taken account of. Nevertheless, when the non-adiabaticity is weak, only the z component of the field strength is essential, which reads

$$F_s^{\mu\nu,z} = \partial^\mu A_s^{\nu,z} - \partial^\nu A_s^{\mu,z} + \Phi^{\mu\nu}, \quad (42)$$

where

$$\Phi^{\mu\nu} \equiv (A_s^{\mu,x} A_s^{\nu,y} - A_s^{\nu,x} A_s^{\mu,y}), \quad (43)$$

is an anomalous field strength representing a trace of the SU(2) gauge field. In terms of a unit vector \mathbf{n} , it reads [33]

$$\Phi^{\mu\nu} = \frac{1}{4} \mathbf{n} \cdot (\partial^\mu \mathbf{n} \times \partial^\nu \mathbf{n}). \quad (44)$$

In the adiabatic limit, $F_s^{\mu\nu,\alpha}$ with spin component $\alpha = x, y$ are suppressed and the z component of the Bianchi identity (41) reduces to

$$\epsilon^{\mu\nu\rho\sigma} \partial_\nu F_{s,\rho\sigma,z} = \epsilon^{\mu\nu\rho\sigma} \partial_\nu (\partial_\rho A_{s,\sigma}^z - \partial_\sigma A_{s,\rho}^z + \Phi_{\rho\sigma}) = 0. \quad (45)$$

From a gauge invariance, the effective electric and magnetic fields are defined as

$$\begin{aligned} E_{s,i} &\equiv F_{s,0i} = -\nabla_i A_{s,0}^z + \partial_t A_{s,i}^z = -\frac{1}{2} \mathbf{n} \cdot (\dot{\mathbf{n}} \times \nabla_i \mathbf{n}) \\ B_{s,i} &= (\nabla \times \mathbf{A}_s^z)_i = \frac{1}{4} \sum_{jk} \epsilon_{ijk} \mathbf{n} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n}). \end{aligned} \quad (46)$$

The $\mu = 0$ component of Eq. (45) then becomes

$$-\nabla \cdot \mathbf{B}_s + \sum_{ijk} \epsilon_{ijk} \partial_i \Phi_{jk} = 0, \quad (47)$$

namely

$$\nabla \cdot \mathbf{B}_s = \rho_h, \quad (48)$$

where

$$\rho_h \equiv \sum_{ijk} \epsilon_{ijk} \partial_i \Phi_{jk} = \frac{1}{4} \sum_{ijk} \epsilon_{ijk} [\partial_i \mathbf{n} \cdot (\partial_j \mathbf{n} \times \partial_k \mathbf{n})]. \quad (49)$$

Thus the effective magnetic field has a finite divergence, i.e., a finite monopole density.

Similarly, $\mu = i$ component of Eq. (45) reads

$$(\nabla \times \mathbf{E}_s)_i + \partial_t \cdot \mathbf{B}_{s,i} = -j_{h,i}, \quad (50)$$

where

$$j_{h,i} \equiv \sum_{jk} \epsilon_{ijk} (\partial_t \Phi_{jk} - 2\partial_j \Phi_{0k}) = \frac{3}{4} \sum_{jk} \epsilon_{ijk} [\dot{\mathbf{n}} \cdot (\partial_j \mathbf{n} \times \partial_k \mathbf{n})]. \quad (51)$$

Consistency of Eqs. (48) (50) is guaranteed by a conservation law for monopole,

$$\dot{\rho}_h + \nabla \cdot \mathbf{j}_h = 0. \quad (52)$$

Therefore, ferromagnetic metals having a singular hedgehog spin structure in the adiabatic limit contains monopole.

C. Quantization condition of hedgehog monopole

An important feature of hedgehog monopole is that its density and current, j_h and ρ_h , vanish when the length of local spin is constant. This is easily seen by noting that a unit vector \mathbf{n} is described by two independent angles, θ and ϕ , and that three vectors $\partial_i \mathbf{n}$, $\partial_j \mathbf{n}$ and $\partial_k \mathbf{n}$ in Eqs. (49) and (51) cannot be independent. Nevertheless, the volume integral of the monopole density is finite due to the surface contribution if the local spin has a hedgehog structure shown in Fig. 4. Let us define a normalized spin gauge field including a coupling constant $\frac{g}{2\pi}$, i.e., as (upper suffix N means north)

$$A_\mu^N \equiv \frac{g}{4\pi} (1 - \cos \theta) \partial_\mu \phi. \quad (53)$$

The effective magnetic field of \mathbf{A}^N reads

$$B_i^N \equiv (\nabla \times \mathbf{A}_\mu^N)_i = \frac{g}{8\pi} \sum_{jk} \epsilon_{ijk} \mathbf{n} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n}). \quad (54)$$

The total magnetic flux is then

$$\int d\mathbf{S} \cdot \mathbf{B}^N = gn, \quad (55)$$

meaning that there are n monopoles with a charge g . Here we used a relation

$$\sum_{ijk} \int dS_i \epsilon_{ijk} \mathbf{n} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n}) = 2 \sum_{ijk} \int dS_i \epsilon_{ijk} \sin \theta (\nabla_j \theta) (\nabla_k \phi) = 8\pi n, \quad (56)$$

where n is an integer. This is understood by noticing that $\sin \theta d\theta d\phi$ is an element of area of a sphere spanned by θ and ϕ , and thus an integral $\sum_{ijk} \int dS_i \epsilon_{ijk} \sin \theta (\nabla_j \theta) (\nabla_k \phi)$ is a solid angle of a sphere (4π) times a winding number of spin structure, an integer n . This relation is a result of a fact that the total solid angle subtended by a spin structure is $4\pi n$, or equivalently a fact that the left-hand side of Eq. (56) is a winding number multiplied by 8π . The hedgehog monopole from the spin structure is thus a topological object having vanishing local densities. The one shown in Fig. 4 has a topological number of $n = 1$.

A quantization of monopole charge g is discussed by requiring that a gauge field covering the whole space without singularity is constructed by patching together locally defined gauge fields. In fact, Eq. (53) is not defined at $\theta = \pi$ (south pole), since $\partial\phi$ can not be defined there. We can define a gauge field regular at the south pole as

$$A_\mu^S = -\frac{g}{4\pi} (1 + \cos \theta) \partial_\mu \phi. \quad (57)$$

This field has a singularity at the north pole $\theta = 0$, but represents the same magnetic field as Eq. (54). We can define therefore only a gauge field with a singularity, if we try to describe the whole space by a single gauge field. The singularity is regarded as a Dirac's string. Instead, we can cover the whole space by patching two gauge fields, A_μ^S and Eq. (53), which we call A_μ^N . They are related by a gauge transformation

$$A_\mu^N = A_\mu^S + i \frac{\hbar}{e} \Theta^{-1} \partial_\mu \Theta, \quad (58)$$

where

$$\Theta \equiv e^{-i \frac{eg}{\hbar} \phi}, \quad (59)$$

is a gauge transform function. This function must be single valued, i.e., is invariant under $\phi \rightarrow \phi + 2\pi$. Thus a condition

$$eg = 2\pi\hbar, \quad (60)$$

is imposed, which is a Dirac's quantization condition (Eq. (15)).

Mechanism of generation of a hedgehog monopole is essentially the same as a GUT monopole, namely, a symmetry breaking of a non-Abelian gauge field (SU(2) for a hedgehog and SU(5) for GUT). There is, however, a difference in the two models. A GUT monopole is a composite object of a gauge field and a Higgs field, and the monopole solution is solved in the same footing as a Higgs field. In a case of hedgehog monopole, in contrast, local spin structure is treated as a background field which is treated as not to be affected by gauge field.

The effective magnetic field of a hedgehog monopole configuration, Eq. (46), is sometimes called the spin Berry's phase, and the electric field is known as the spin motive force [8]. The magnetic field has been observed for example in the anomalous Hall effect [17], and the electric field has been observed by inducing magnetization dynamics such as domain wall motion [15]. These experiments do not, however, mean existence of hedgehog monopole, since as explained in §IV.

D. Hedgehog monopole in weak exchange coupling regime

Until very recently, a hedgehog monopole has been discussed exclusively in the adiabatic (strong sd coupling) regime. As we have discussed in the earlier sections, a symmetry breaking of spin SU(2) space to a U(1) space of electromagnetism is clearly defined in this regime. However, hedgehog monopole is not an object restricted to adiabatic regime. In fact, introduction of an exchange interaction to rotationally invariant spin results in a symmetry breaking even if the interaction is weak. In this section, we will demonstrate that the hedgehog monopole emerges even if the exchange coupling J is small following the analysis in Ref. [35]. We cannot approach monopoles in this regime by a standard gauge field argument. Here we apply instead a novel method pointed out in Ref. [10, 35] based on a transport calculation. In a transport method, effective electromagnetic fields are calculated by evaluating electric charge density and current density induced by local spin structure by use of Keldysh Green's functions. A monopole field is then identified by deriving the Maxwell's equation for the effective fields.

We consider a disordered metal, and take account of the spin-independent impurity scat-

tering represented as

$$H_i = \int d^3r c^\dagger v_i c, \quad (61)$$

v_i being the impurity potential. In the following calculation, the impurities are approximated as random point scatterers and the averaging is carried out as

$$\langle v_i(\mathbf{r}) v_i(\mathbf{r}') \rangle_i = n_i u_i^2 \delta^3(\mathbf{r} - \mathbf{r}'), \quad (62)$$

where n_i and u_i are the impurity concentration and the strength of the scattering, respectively [36]. The impurities give rise to an elastic lifetime for the electron, τ , which is calculated as

$$\tau = \frac{\hbar}{2\pi n_i u_i^2 \nu}, \quad (63)$$

(ν is the density of states per volume). The total Hamiltonian discussed in this section is

$$H = H_0 + H_i + H_{sd}. \quad (64)$$

The electric charge density is

$$\rho(\mathbf{r}, t) = -e \text{tr} \langle c^\dagger(\mathbf{r}, t) c(\mathbf{r}, t) \rangle, \quad (65)$$

where the bracket represents the quantum expectation value and tr is a trace over spin indices, and electric current density is given as

$$j_i(\mathbf{r}, t) = \frac{e\hbar^2}{2m} (\nabla_{\mathbf{r}} - \nabla_{\mathbf{r}'})_i \text{tr} G^<(\mathbf{r}, t; \mathbf{r}', t) \big|_{\mathbf{r}'=\mathbf{r}}, \quad (66)$$

where

$$G_{s,s'}^<(\mathbf{r}, t; \mathbf{r}', t') \equiv \frac{i}{\hbar} \langle c_{s'}^\dagger(\mathbf{r}', t') c_s(\mathbf{r}, t) \rangle, \quad (67)$$

(s and s' are spin indices) is the lesser component of the non-equilibrium Green's function [37]. This Green's function defined on the Keldysh contour C satisfies the Dyson's equation,

$$\begin{aligned} G_{ss'}(\mathbf{r}, t; \mathbf{r}', t') &= \delta_{s,s'} g_s(\mathbf{r}, t; \mathbf{r}', t') \\ &+ \int d^3r'' \int_C dt'' g_s(\mathbf{r}, t; \mathbf{r}'', t'') \\ &\times [\delta_{s,s''} v_i(\mathbf{r}'') - J \mathbf{S}(\mathbf{r}'', t'') \cdot \boldsymbol{\sigma}_{ss''}] \\ &\times G_{s''s'}(\mathbf{r}'', t''; \mathbf{r}', t'), \end{aligned} \quad (68)$$

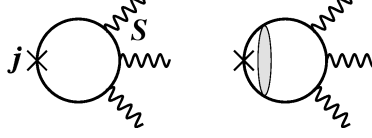


FIG. 9. Diagrammatic representation of the electric current, \mathbf{j} , induced by sd interaction with local spin, \mathbf{S} . Solid lines represent the conduction electron's Green's function and wavy lines are the interaction with \mathbf{S} . The first diagram is a contribution of an effective electric and magnetic fields local in space, and the second diagram, containing a diffusion ladder (vertex corrections) denoted by the gray shaded oval, results in a diffusive current (a gradient of electric charge density).

where $G_{ss'}(\mathbf{r}, t; \mathbf{r}', t') \equiv -(i/\hbar) \langle T_C [c_s(\mathbf{r}, t) c_{s'}^\dagger(\mathbf{r}', t')] \rangle$ (T_C is the path-ordering operator on C) and g denotes free Green's function. This equation is solved by iteration. Here, we assume slowly varying magnetization profile $\mathbf{S}_{\mathbf{q}, \Omega}$ in space and time: the spatially smooth magnetization structure compared to the electron mean free path ℓ , $q\ell \ll 1$ (q is a wave number of magnetization texture), and the sufficiently slow dynamics of magnetization, $\Omega\tau \ll 1$ (Ω is a frequency of magnetization dynamics).

To see a hedgehog monopole, it is enough to discuss the electric current to the third-order in the exchange coupling, J . This contribution is diagrammatically shown in Fig. 9 and is calculated as

$$j_i(\mathbf{r}, t) = \frac{eJ^3}{\pi m V} \sum_{\mathbf{k}, \mathbf{q}, \mathbf{q}', \mathbf{Q}} \sum_{\omega, \omega', \Omega} e^{-i\mathbf{Q} \cdot \mathbf{r} + i\Omega t} \mathbf{S}_{\mathbf{q}, \omega} \cdot (\mathbf{S}_{\mathbf{q}', \omega'} \times \mathbf{S}_{\mathbf{Q}-\mathbf{q}-\mathbf{q}', \Omega-\omega-\omega'}) \\ \times \left[\frac{i\hbar^3}{30m} q_i (\mathbf{Q} \cdot \mathbf{q}') \text{Im}(g_{\mathbf{k}}^a)^4 - 2\tau^2 \omega q_i' |g_{\mathbf{k}}^a|^2 \right] - D \nabla_i \rho(\mathbf{r}, t), \quad (69)$$

where

$$g_{\mathbf{k}}^a = \frac{1}{\epsilon_F - \frac{\hbar^2 \mathbf{k}^2}{2m} - \frac{i\hbar}{2\tau}}, \quad (70)$$

is the advanced Green's function (ϵ_F is the Fermi energy), Im means taking an imaginary component, V is system volume, and $D = 2\epsilon_F \tau / 3m$ denotes a diffusion constant. The last term is a diffusive contribution arising from vertex corrections shown in the right diagram of Fig. 9 and electric charge density, ρ , is calculated as

$$\rho = \frac{4e\nu J^3 \tau^4}{\hbar m} \nabla_i \left\langle \mathbf{S} \cdot (\dot{\mathbf{S}} \times \nabla_i \mathbf{S}) \right\rangle_D, \quad (71)$$

where $\langle \cdot \cdot \rangle_{\text{D}}$ is an average including electron diffusion, which satisfies

$$(-D\nabla^2 + \partial_t)\langle F(\mathbf{r}, t) \rangle_{\text{D}} = \frac{1}{\tau}F(\mathbf{r}, t), \quad (72)$$

(F is an arbitrary function depending on space \mathbf{r} and time t).

Summing over the wave vectors in Eq. (69), we obtain the electric current,

$$j_i = -\frac{e\hbar^3\nu J^3}{960m^2\epsilon_F^3}\epsilon_{ijk}\epsilon_{klm}\nabla_j[\mathbf{S} \cdot (\nabla_l\mathbf{S} \times \nabla_m\mathbf{S})] - \frac{4e\nu J^3\tau^3}{\hbar m}\mathbf{S} \cdot (\dot{\mathbf{S}} \times \nabla_i\mathbf{S}) - D\nabla_i\rho. \quad (73)$$

As was pointed out in Ref. [35], the effective electric and magnetic fields (\mathbf{E}_{h} and \mathbf{B}_{h}) are read from the above result by comparing it with a general expression,

$$\mathbf{j} = (1/\mu)\nabla \times \mathbf{B}_{\text{h}} + \sigma_{\text{c}}\mathbf{E}_{\text{h}} - D\nabla\rho, \quad (74)$$

where μ is magnetic permeability,

$$\sigma_{\text{c}} \equiv e^2 n\tau/m = \frac{e^2}{3} \frac{\nu}{V} \left(\frac{\hbar k_F}{m} \right)^2 \tau, \quad (75)$$

is electric conductivity. The result is

$$E_{\text{h},i} = -2\gamma_{\text{h}}\mathbf{S} \cdot (\dot{\mathbf{S}} \times \nabla_i\mathbf{S}), \quad (76)$$

$$B_{\text{h},i} = \gamma_{\text{h}}\epsilon_{ijk}\mathbf{S} \cdot (\nabla_j\mathbf{S} \times \nabla_k\mathbf{S}), \quad (77)$$

where

$$\gamma_{\text{h}} \equiv 6\hbar \frac{mJ^3\tau^2}{\hbar^4 k_F^2}, \quad (78)$$

and we defined magnetic permeability as

$$\frac{1}{\mu} = -\frac{1}{45 \cdot 2^5} \frac{e^2}{\hbar^2 k_F^4} \frac{\nu}{V} \left(\frac{\hbar}{\tau} \right)^2. \quad (79)$$

Obviously, these effective fields satisfy the Faraday's law and the Gauss's law with magnetic monopole,

$$\begin{aligned} \nabla \times \mathbf{E}_{\text{h}} + \dot{\mathbf{B}}_{\text{h}} &= -\mathbf{j}_{\text{h}} \\ \nabla \cdot \mathbf{B}_{\text{h}} &= \rho_{\text{h}}, \end{aligned} \quad (80)$$

where magnetic monopole contributions are

$$j_{\text{h},i} = -3\gamma_{\text{h}}\epsilon_{ijk}\dot{\mathbf{S}} \cdot (\nabla_j\mathbf{S} \times \nabla_k\mathbf{S}), \quad (81)$$

$$\rho_{\text{h}} = \gamma_{\text{h}}\epsilon_{ijk}\nabla_i\mathbf{S} \cdot (\nabla_j\mathbf{S} \times \nabla_k\mathbf{S}). \quad (82)$$

There is therefore hedgehog monopole in the weak sd coupling case, too, although the coefficients in Eq. (82) differ from the strong coupling limit. It is notable that the structure of the electromagnetism, i.e., U(1) gauge field, (and further, that with monopole in the present case) is embedded in the electron transport phenomena.

We note here that the definition of \mathbf{E}_h , \mathbf{B}_h , μ according to Eq. (74) and the dielectric constant ϵ according to the Gauss's law has arbitrariness. In fact, the condition imposed by the transport properties (\mathbf{j} and ρ) is not sufficient to fix the two effective fields uniquely, and an additional condition seems to be required. Nevertheless, existence of monopole holds true; both of the monopole current and density cannot be deleted at the same time by redefining the fields. Further, as noted in Ref. [35], the product $\epsilon\mu$ is invariant; a physical quantity of the velocity of the effective topological electromagnetic field is uniquely given as

$$v_{\text{top}} \equiv \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{8\sqrt{30}} \frac{\hbar}{k_F \epsilon_F \tau^2}. \quad (83)$$

If elastic mean free path, $\ell \equiv \frac{\hbar k_F \tau}{m}$, is 10\AA , $\epsilon_F \tau / \hbar = 3.3$ by choosing $k_F^{-1} = 1.5\text{\AA}$, the speed of the topological electromagnetic wave is thus rather large, about 900m/s . The topological electromagnetic field may be useful to transport spin information in a different manner from magnon transports.

VI. SPIN DAMPING MONOPOLE

As we have seen, the hedgehog monopole is a topological object and has locally vanishing density and current only. The hedgehog monopole hence does not locally coupled to the electromagnetism. Very recently, a novel monopole in magnets with locally finite density and current density was discovered by Takeuchi et al [10]. Such a monopole creates a rotational electric field via the Ampère's law (Fig. 2), and thus it acts as an anomalous angular momentum source which induces rotational motion of electric charge. To realize such a monopole, Takeuchi et al. included the spin-orbit interaction.

The spin-orbit interaction exists in any elements including magnetic ones, and is particularly strong in heavy elements such as platinum and gold, and at the interfaces in junctions where the inversion symmetry is broken [38]. Two types of the spin-orbit interaction were thus considered in Ref. [10]. The first is the one from a uniform field, \mathbf{E}_R , namely the Rashba interaction [39]. Such a field is realized at the interfaces and surfaces. The Rashba

interaction in metallic films recently turned out to be particularly useful for current-driven magnetization switching [40, 41]. The second is the one from a random potential, v_i , induced by heavy impurities. This random heavy impurity model would also simulate the effect of spin-orbit interaction in heavy pure metals. The total spin-orbit interaction thus reads

$$H_{\text{so}} = -\frac{1}{\hbar} \int d^3r c^\dagger [(\lambda_R \mathbf{E}_R - \lambda_i \nabla v_i) \cdot (\mathbf{p} \times \boldsymbol{\sigma})] c, \quad (84)$$

where \mathbf{p} is the electron's momentum and λ is a spin-orbit coupling constant (the subscript R and i characterize Rashba and impurity-induced ones, respectively). The interaction with the magnetization is described by H_{sd} . The Hamiltonian of the present system is, therefore, given as

$$H = H_0 + H_i + H_{\text{sd}} + H_{\text{so}}. \quad (85)$$

The electric current, \mathbf{j} , generated in the system by magnetization dynamics is calculated by evaluating a quantum field theoretical expectation value of electron velocity operator,

$$\hat{\mathbf{v}} = -(i\hbar/m)\nabla + (1/\hbar)(\lambda_R \mathbf{E}_R - \lambda_i \nabla v_i) \times \boldsymbol{\sigma}. \quad (86)$$

The electric current thus reads

$$\mathbf{j}(\mathbf{r}, t) = e \text{tr} \left[\left(\frac{\hbar^2}{2m} (\nabla_{\mathbf{r}} - \nabla_{\mathbf{r}'}) + i[\lambda_R \mathbf{E}_R - \lambda_i \nabla v_i(\mathbf{r})] \times \boldsymbol{\sigma} \right) G^<(\mathbf{r}, t; \mathbf{r}', t) \right]_{\mathbf{r}'=\mathbf{r}}. \quad (87)$$

This expectation value is evaluated by solving the following Dyson's equation,

$$\begin{aligned} G_{ss'}(\mathbf{r}, t; \mathbf{r}', t') &= \delta_{s,s'} g_s(\mathbf{r}, t; \mathbf{r}', t') \\ &+ \int d^3r'' \int_C dt'' g_s(\mathbf{r}, t; \mathbf{r}'', t'') \\ &\times (\delta_{s,s''} v_i(\mathbf{r}'') - J \mathbf{S}(\mathbf{r}'', t'') \cdot \boldsymbol{\sigma}_{ss''}) \\ &+ i\{[\lambda_R \mathbf{E}_R - \lambda_i \nabla v_i(\mathbf{r}'')] \times \nabla_{\mathbf{r}''}\} \cdot \boldsymbol{\sigma}_{ss''}) \\ &\times G_{s''s'}(\mathbf{r}'', t''; \mathbf{r}', t'). \end{aligned} \quad (88)$$

This equation is solved treating λ and J perturbatively to the linear and second orders, respectively. Contributions to current is represented by Feynman diagrams in Figs. 10 and 11.

We consider sufficiently slow dynamics of magnetization, namely $\Omega\tau \ll 1$ (Ω is a frequency of magnetization dynamics), and assume that the magnetization structure varies smoothly

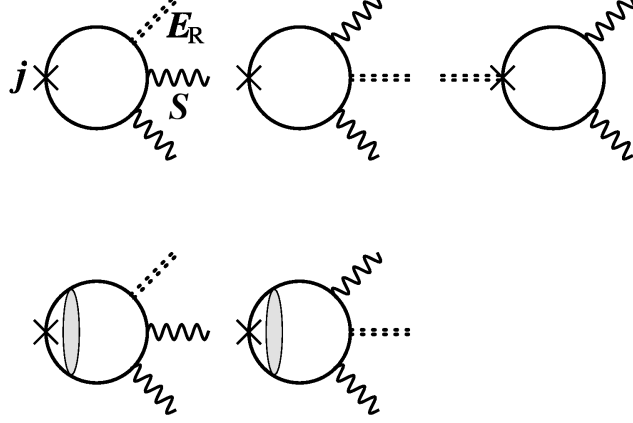


FIG. 10. Diagrammatic representations of electric current pumped by magnetization dynamics and the Rashba interaction. Solid lines represent the conducting electron Green's functions, and double dashed and wavy lines represent the Rashba spin-orbit interaction (\mathbf{E}_R) and the interaction with localized spin (\mathbf{S}), respectively. The first three contributions correspond to effective electromagnetic fields, and the last two contributions, containing diffusion ladders (vertex corrections) denoted by the gray shaded ovals.

in the space compared to the electron mean free path ℓ , i.e., $q\ell \ll 1$ (q is a wave number of magnetization profile). The leading contribution in this case turns out to be

$$\begin{aligned}
 \mathbf{j}(\mathbf{r}, t) = & -\frac{eJ^2}{2\pi V} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}_1, \mathbf{q}_2} \sum_{\omega, \Omega_1, \Omega_2} e^{-i(\mathbf{q}_1 + \mathbf{q}_2) \cdot \mathbf{r} + i(\Omega_1 + \Omega_2)t} \Omega_1 \\
 & \times (\mathbf{S}_{\mathbf{q}_1, \Omega_1} \times \mathbf{S}_{\mathbf{q}_2, \Omega_2}) \times \left[\frac{i\lambda_R \tau}{\hbar} \mathbf{E}_R |g_{\mathbf{k}}^a|^2 \right. \\
 & + \frac{4\hbar^2 \lambda_i}{3\pi \nu \tau^2} (\mathbf{q}_1 + \mathbf{q}_2) \varepsilon_{\mathbf{k}} |g_{\mathbf{k}}^a|^2 |g_{\mathbf{k}'}^a|^4 \left. \right] \\
 & - D \nabla \rho(\mathbf{r}, t).
 \end{aligned} \tag{89}$$

The last term is the diffusive contribution arising from the vertex corrections, where the electric charge density ρ is

$$\rho = \frac{4e\nu\lambda_R J^2 \tau^3}{\hbar^2} \nabla \cdot \langle \mathbf{E}_R \times (\mathbf{S} \times \dot{\mathbf{S}}) \rangle_D. \tag{90}$$

Here $\langle \cdots \rangle_D$ represents the average including the electron diffusion, defined in Eq. (72). Summing over the wave vectors and frequencies in Eq. (89), the electric current is obtained

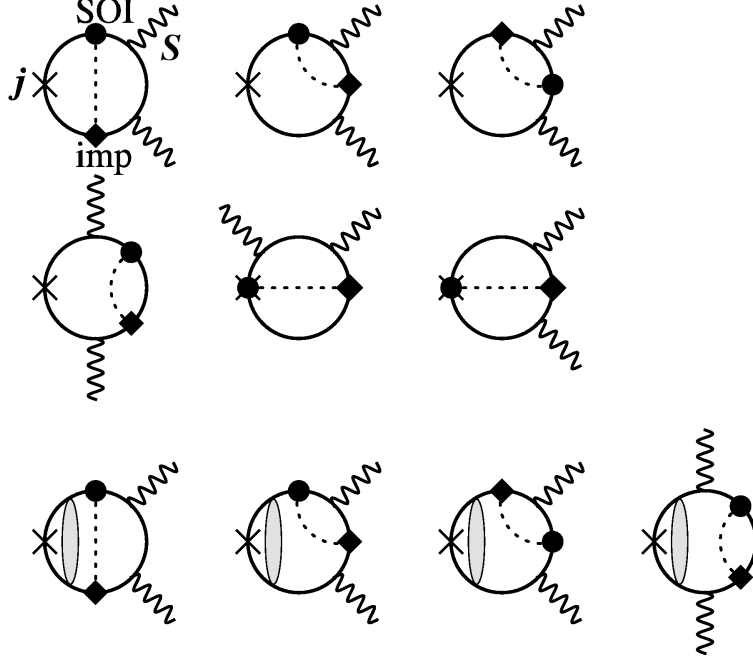


FIG. 11. Contribution to the electric current driven by a precession of localized spin and random impurity-induced spin-orbit interaction. Filled circles are spin-orbit interactions arising from random impurities (SOI), filled diamonds are nonmagnetic impurity scatterings (imp), and dotted lines linking filled circles to filled diamonds represent impurity average.

as

$$\begin{aligned} \mathbf{j} = & -\frac{16e\nu\lambda_i J^2 \epsilon_F \tau^2}{3\hbar^2} \nabla \times (\mathbf{S} \times \dot{\mathbf{S}}) \\ & - \frac{4e\nu\lambda_R J^2 \tau^2}{\hbar^2} \mathbf{E}_R \times (\mathbf{S} \times \dot{\mathbf{S}}) - D \nabla \rho. \end{aligned} \quad (91)$$

This result is rewritten by use of effective electric and magnetic fields, \mathbf{E}_s and \mathbf{B}_s , as

$$\mathbf{j} = \frac{1}{\mu} \nabla \times \mathbf{B}_s + \sigma_c \mathbf{E}_s - D \nabla \rho, \quad (92)$$

where the effective fields are defined as

$$\begin{aligned} \mathbf{E}_s & \equiv -\alpha_R \mathbf{E}_R \times \mathbf{N}, \\ \mathbf{B}_s & \equiv -\beta_i \mathbf{N}. \end{aligned} \quad (93)$$

Here

$$\mathbf{N} \equiv \mathbf{S} \times \dot{\mathbf{S}}, \quad (94)$$

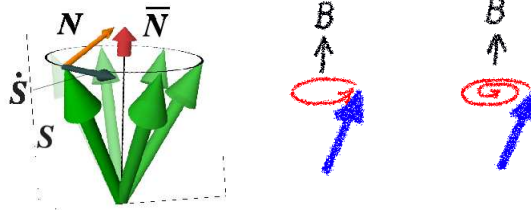


FIG. 12. Left: Schematic illustration of damping of a precessing local spin \mathbf{S} . A vector product of a time derivative of spin, $\dot{\mathbf{S}}$, and \mathbf{S} is a vector $\mathbf{N} \equiv \mathbf{S} \times \dot{\mathbf{S}}$. Its average is $\bar{\mathbf{N}}$, which is along the precession axis, and it represents a time-averaged dissipated spin magnitude. Right: Damping torque, \mathbf{N} , results in a damping of spin precession when an external magnetic field \mathbf{B} is applied.

is a vector representing the spin damping torque (see §B) (Fig. 12) [33, 42]. Coefficients α_R and β_i are

$$\begin{aligned}\alpha_R &\equiv \frac{4e\nu\lambda_R J^2 \tau^2}{\sigma_c \hbar^2} \\ \beta_i &\equiv \frac{16e\nu\mu\lambda_i J^2 \epsilon_F \tau^2}{3\hbar^2}.\end{aligned}\tag{95}$$

The effective fields calculated here are the ones acting on the electronic spin in the same manner as the effective fields from the hedgehog monopole. Clearly, the fields [Eq. (93)] do not satisfy the Faraday's law and the Gauss's law of the conventional electromagnetism, but the ones with monopole contribution,

$$\begin{aligned}\nabla \times \mathbf{E}_s + \dot{\mathbf{B}}_s &= -\mathbf{j}_m, \\ \nabla \cdot \mathbf{B}_s &= \rho_m,\end{aligned}\tag{96}$$

where the monopole current and monopole density read

$$\mathbf{j}_m = \alpha_R \nabla \times (\mathbf{E}_R \times \mathbf{N}) + \beta_i \dot{\mathbf{N}},\tag{97}$$

and

$$\rho_m = -\beta_i \nabla \cdot \mathbf{N}.\tag{98}$$

We have thus proved that a monopole emerges when spin damping occurs, namely we have a spin damping monopole. The spin damping monopole is a composite object made from a magnetization configuration in the same manner as the hedgehog monopole. The monopole satisfies the conservation law, $\dot{\rho}_m + \nabla \cdot \mathbf{j}_m = 0$.

A. Remarks on uniqueness of effective fields

Equation (96) apparently contains an arbitrariness. In fact, one may think that a transformation $\mathbf{E}_s \rightarrow \mathbf{E}_s + (1/\sigma_c)\nabla \times \mathbf{C}$ and $\mathbf{B}_s \rightarrow \mathbf{B}_s - \mu\mathbf{C}$, where \mathbf{C} is an arbitrary vector field, is possible without changing Eq. (96). However, such a transform is not allowed because of a gauge invariance in the original space with a higher symmetry, as is known in the case of hedgehog monopole, where an SU(2) gauge invariance forbids this arbitrariness.

Validness of our definition of \mathbf{E}_s and \mathbf{B}_s in Eq. (93) is supported by the following argument. In Eq. (92), electron diffusion ($D\nabla\rho$) induces the effective electric and magnetic polarizations \mathbf{P}_s and \mathbf{M}_s , respectively. By using the relation

$$(-D\nabla^2 + \partial_t)\rho = -\sigma_c \nabla \cdot \mathbf{E}_s, \quad (99)$$

the electric current (eq. (92)) is described as the rotation of the magnetic field and the time derivative of the electric field,

$$\mathbf{j} = \nabla \times \mathbf{H}_s - \dot{\mathbf{D}}_s, \quad (100)$$

where \mathbf{H}_s and \mathbf{D}_s are the fields defined as

$$\begin{aligned} \mathbf{H}_s &\equiv \frac{1}{\mu} \mathbf{B}_s - \mathbf{M}_s \\ \mathbf{D}_s &\equiv \varepsilon \mathbf{E}_s + \mathbf{P}_s, \end{aligned} \quad (101)$$

with $\varepsilon = -\sigma_c\tau$ being the permittivity, respectively. In Eq. (101), \mathbf{E}_s and \mathbf{B}_s are naturally identified with the electric and magnetic fields, respectively. By use of these fields, two of the Maxwell's equations become

$$\begin{aligned} \nabla \times \mathbf{D}_s + \dot{\mathbf{H}}_s &= \mathbf{j}'_m \\ \nabla \cdot \mathbf{H}_s &= \rho_m. \end{aligned} \quad (102)$$

Here \mathbf{j}'_m is redefined monopole current, which is non-local.

B. Spin damping monopole generation in a ferro-normal junction

The spin damping monopole is unique since it does not require a particular non-coplanar spin structure like a hedgehog, and so it exists quite generally in magnetic systems. The

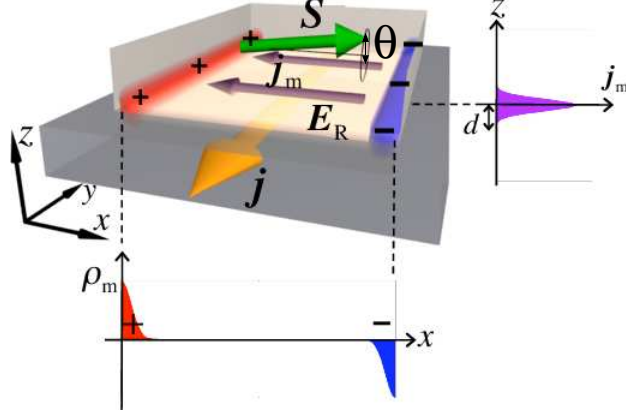


FIG. 13. Schematic illustration of monopole pumping and detection in a thin ferromagnetic film attached on a non-magnetic layer. Magnetization (\mathbf{S}) precession is induced by applying an oscillating magnetic field. The amplitude of precession is represented by an angle θ . The Rashba field, \mathbf{E}_R , exists at the interface and creates the monopole current, \mathbf{j}_m , near the interface. The width of the monopole current distribution, d , is comparable to the decay length of the magnetization at the interface. The monopole current induces an electric current, \mathbf{j} , via the Ampère's law at the interface. The impurity spin-orbit interaction directly induces positive (+) and negative (-) monopole charge distribution, ρ_m , at the two edges.

simplest candidate for creating the monopole would be a thin ferromagnetic film put on a non-magnetic insulator or metal shown in Fig. 13. We choose the z axis perpendicular to the film. The Rashba-type spin-orbit field would then arise at the interface along the z direction [38]. We excite the precession of the uniform magnetization by applying the alternating magnetic field in the yz plane in the presence of static field along the x axis (ferromagnetic resonance [42]). The precession results in the spin damping vector with a finite time average, $\overline{\mathbf{N}}$, along x direction. In the present case with the uniform magnetization, spatial derivatives in Eqs. (97) and (98) arise at the interface and at the edges, where the magnetization vanishes. The Rashba interaction contributes to the DC monopole current at the interface as

$$\overline{j_{m,x}^R} = -\alpha_R E_R \frac{\partial \overline{N}}{\partial z} \simeq -\frac{\alpha_R}{d} E_R \overline{N}, \quad (103)$$

where d is the spatial scale of the magnetization decay at the interface. The monopole current driven by the random spin-orbit impurities, on the other hand, vanishes when time-

averaged. The total DC monopole current thus reads

$$\overline{\mathbf{j}}_{\text{m}} = -\mathbf{e}_x \frac{\alpha_{\text{R}}}{d} E_{\text{R}} \overline{N}, \quad (104)$$

(\mathbf{e}_x represents the unit vector along the x direction). This monopole current at the interface generates the electromotive force along the y direction via the Ampère's law for the monopole. The monopole density induced by the random spin-orbit interaction arises at the edge of the ferromagnetic film since $\nabla \cdot \mathbf{N} \simeq \partial N_x / \partial x$ is finite there. The induced monopole density at the two edges is

$$\overline{\rho}_{\text{m}} = \mp \frac{\beta_{\text{i}}}{d} \overline{N}, \quad (105)$$

where the sign is positive on one side of the edge and negative on the other side. The monopoles then produces a magnetic field along the x direction as

$$\overline{\mathbf{B}}_{\text{s}} = -\mathbf{e}_x \beta_{\text{i}} \overline{N}. \quad (106)$$

This field creates the electric current in the y direction via the conventional Ampère's law. The averaged electric current density generated by the spin damping monopole [Eq. (92)] thus reduces to

$$\overline{\mathbf{j}} = -\mathbf{e}_y \left(\sigma_{\text{c}} \alpha_{\text{R}} E_{\text{R}} + \frac{\beta_{\text{i}}}{\mu d} \right) \overline{N}. \quad (107)$$

We have thus found that when magnetization precession occurs in a metallic ferro-normal junction, a current of voltage arises perpendicular to the junction and a precession axis. As far as electric detection concerns, a monopole effect appears qualitatively the same as an inverse spin Hall effect [12], which is a widely-used experimental method for spin current detection. We will examine this important point later in §VII A.

Let us look into the monopole signal quantitatively. We define energy scales of Rashba and impurity spin-orbit interactions as (k_{F} is Fermi wavelength)

$$\begin{aligned} \Delta_{\text{R}} &\equiv \frac{\lambda_{\text{R}}}{e} k_{\text{F}}^2 v_{\text{R}} \\ \Delta_{\text{i}} &\equiv \lambda_{\text{i}} k_{\text{F}}^2 v_{\text{i}}, \end{aligned} \quad (108)$$

where $v_{\text{R}} \equiv \frac{e}{k_{\text{F}}} E_{\text{R}}$ is a potential energy due to Rashba electric field, \mathbf{E}_{R} . Coefficients α_{R} and β_{i} then read

$$\begin{aligned} \alpha_{\text{R}} &= 12\pi^2 \left(\frac{J}{\epsilon_{\text{F}}} \right)^2 \left(\frac{\Delta_{\text{R}}}{v_{\text{R}}} \right) \tau \\ \beta_{\text{i}} &= \frac{16}{3} \mu e k_{\text{F}} \left(\frac{J}{\epsilon_{\text{F}}} \right)^2 \left(\frac{\Delta_{\text{i}}}{v_{\text{i}}} \right) \left(\frac{\epsilon_{\text{F}} \tau}{\hbar} \right)^2. \end{aligned} \quad (109)$$

β_i and $\mu e k_F$ have dimension of Tesla·sec. Here we used $\nu = k_F^3/\epsilon_F$, $n = k_F^3/(6\pi^2)$ and $\sigma_c = e^2 n \tau / m$.

When the spin damping arises from the magnetization precession with the frequency Ω and the angle θ (Fig. 13), the magnitude of a monopole-induced current density, Eq. (107), reads

$$|\bar{j}| = 4e k_F^2 \Omega \left(\frac{J\tau}{\hbar} \right)^2 \sin \theta \left[\frac{\Delta_R}{\epsilon_F} + \frac{4}{3} \frac{1}{k_F d} \frac{\Delta_i}{v_i} \right]. \quad (110)$$

We consider a disordered ferromagnets with $J/\epsilon_F \sim 0.1$, $\epsilon_F \tau / \hbar \sim 10$ and $k_F^{-1} \sim 2 \text{ \AA}$ ($\epsilon_F = \frac{\hbar^2 k_F^2}{2m} = 1.51 \times 10^{-19} \text{ J} = 0.95 \text{ eV}$ and $\tau = 7.0 \times 10^{-15} \text{ s}$). Rashba interaction strength is chosen as $\Delta_R/\epsilon_F \sim 0.1$, considering an enhancement on the surfaces and the interfaces [43]. We assume for simplicity that $\frac{1}{k_F d} \frac{\Delta_i}{v_i}$ is the same order of magnitude as $\frac{\Delta_R}{\epsilon_F}$. When $\theta = 30^\circ$ and $\Omega = 1 \text{ GHz}$, the electric current density is thus

$$|\bar{j}| = 1.6 \times 10^9 \text{ A/m}^2, \quad (111)$$

which is sufficiently large for experimental detection. In addition to DC, there is AC component in Eq. (92) which would be accessible by a time-resolved measurement.

The current corresponds to an effective electric field, Eq. (93), with a magnitude

$$|E_S| = \frac{1}{e} \alpha_R k_F v_R \Omega = 12\pi^2 \frac{\epsilon_F k_F}{e} \left(\frac{\Delta_R}{\epsilon_F} \right)^2 \left(\frac{J}{\epsilon_F} \right)^2 \tau \Omega. \quad (112)$$

For the above values of parameters, the field is 390 V/m. The voltage induced by monopole when a sample width is 100nm is thus $39\mu\text{V}$. The magnitude of effective magnetic field, Eq. (93), is

$$|B_S| = \beta_i \Omega = \frac{16}{3} \mu e k_F \Omega \left(\frac{\Delta_i}{v_i} \right)^2 \left(\frac{J\tau}{\hbar} \right)^2, \quad (113)$$

which is estimated as 5.4m Tesla if we choose $\mu = 10^5 \times \mu_0$ ($\mu_0 = 4\pi \times 10^7 \text{ H/m}$ is permeability in vacuum) as in the case of permeability of permalloy for the ordinary magnetic field. We note that μ here is a permeability for effective magnetic field and can be different from the one for ordinary magnetic field. For correct quantitative estimate, magnetic properties for the effective magnetic field needs to be investigated further.

The monopole current, Eq. (97), is estimated to be

$$|j_m| \simeq \frac{|E_S|}{d} \sim 4 \times 10^{11} \text{ V/m}^2. \quad (114)$$

This value, however, cannot be converted into usual dimension of current density, A/m², since permeability of the effective field is not known.

We note that the electric current estimated here is an initial current that arises when the pumping of monopoles starts. When the monopole current is pumped steadily, the monopole accumulation grows at the edges of the system, inducing a diffusive current. The steady monopole distribution is then determined by the balance of this backward diffusion and the pumped monopole current.

VII. DISCUSSION

In this paper, we have succeeded in proving existence of monopoles by deriving Maxwell's equation by a transport calculation in §VD and §VI. There remains, however, a few points to be clarified in the transport approach to spin damping monopole.

First is a question whether spin damping monopole has a topological meaning or not. In a case of hedgehog monopole, it was a topological object in three space dimensions, while topological nature seems to be lacking in the expression of spin damping monopole density (Eq. (98)). What is crucially different in spin damping monopole is that it needs dynamic spins ($\dot{\mathbf{S}}$). Thus, if there is any topological meaning, it should be discussed in both space and time.

Second point is a gauge field representation of spin damping monopole. In a case of hedgehog monopole, it was obvious in the strong coupling limit. Even in the weak coupling limit, it is straightforward to see from the effective fields (Eq. (77)) that the gauge field describing hedgehog monopole is $\mathbf{A}_S^z = \frac{1}{2}(1 - \cos \theta)\partial_\mu \phi$ (defined in Eq. (33)). A quantization condition for a monopole charge then arose naturally from the condition that a gauge field can be defined to cover the whole space (Eq. (60)). In a case of spin damping monopole, an effective magnetic field in Eq. (93) cannot be written as a rotation of a local effective vector potential (because there is a monopole), but is represented by a nonlocal vector potential as $\mathbf{B}_s = \nabla \times \mathbf{A}^{(\text{sd})} + \mathbf{\Phi}$, where (see Eq. (8))

$$A_i^{(\text{sd})}(\mathbf{r}) = -\beta_i \sum_{jk} \epsilon_{ijk} \int_L dr'_j N_k(\mathbf{r} - \mathbf{r}'), \quad (115)$$

and $\mathbf{\Phi}$ represents a string singularity field. Based on this expression, the same argument as Dirac for a quantization condition [1] is valid, and thus the charge of a spin damping

monopole g is quantized as in Eq. (15). In order to understand fully the origin of spin damping monopole, it is important to study if the monopole is represented by patching two or more local gauge fields (see §V C). This would be carried out by studying a clean ($\tau = \infty$) and strong sd coupling limit.

A. Monopole interpretation of the inverse spin Hall effect

The electromotive force discussed above is a result of the Maxwell's equation for the monopole, which is an exact equation required by the gauge invariance. If one detects the electromotive force given by Eq. (112), it becomes a direct evidence of spin damping monopole. The electromotive force here acts on the spin of the electron, and thus can be detected electrically as has been demonstrated [15]. Remarkably, the observation of spintronics monopoles might have already been achieved. In fact, the electric voltage due to the magnetization precession has been observed in a system of ferromagnet on a Pt film [12] and the direction of the voltage was in agreement with our prediction from the monopole. (The spin-orbit interaction expected in Pt is an intrinsic one due to periodic atoms, but the effect is expected to be the same as the one from the random potential. In addition, interface Rashba interaction might also be there. Therefore the monopole scenario would apply to the system in Ref. [12].) In Ref. [12], the mechanism for the voltage generation was argued to be the inverse spin Hall effect. According to the inverse spin Hall explanation, the magnetization precession generates spin current via the spin pumping effect, and the spin current, j_s , is converted into charge current by the spin-orbit interaction (the inverse of the spin Hall effect). This explanation assumes that the conversion mechanism of [44]

$$j_i = \lambda \sum_{jk} \epsilon_{ijk} j_{s,j}^k, \quad (116)$$

where λ is a constant representing the strength of the spin-orbit interaction and k is the index for the spin polarization of the spin current. This formula is, however, physically incorrect. In fact, the charge current is a conserved quantity, while the spin current is not because of the spin relaxation, and thus these two currents should not be simply proportional to each other. To put in other words, spin current does not have a unique definition because of its relaxation, and thus is not physical. A spin current explanation is therefore an approximate one which might be justified only at very short distance (less than the spin relaxation length).

Indeed, it has been theoretically demonstrated that that conversion formula is not satisfied in a case of slowly varying magnetization configuration [45]. In contrast to the spin current scenario, our monopole scenario is a result of Maxwell's equations, required by a symmetry of the electromagnetism. Since monopole is a conserved quantity satisfying $\dot{\rho}_m + \nabla \cdot \mathbf{j}_m = 0$, monopole scenario is based on a relation between physical quantities, the electric field and monopole. Our scenario therefore explains the "inverse spin Hall effect" free from ambiguity, in contrast to the spin current one.

For experimental confirmation of the spin damping monopole, of crucial importance is the separation of the monopole signal from the inverse spin Hall signal driven by the spin current. This is accomplished by applying an effective electric field perpendicular to the junction of Fig. 13. The monopole contribution then leads to the transverse electric current as a result of the Hall effect for the monopole, while the contribution of the spin current is not affected.

Observation of the effective magnetic field emitted by monopoles, Eq. (93), by use of muons or neutrons or electron holography would be also a direct evidence of monopole generation.

B. Remarks on spin current

Our analysis has also proved that the spin current does not modify the fundamental law of electromagnetism, namely Maxwell's equations. Historically, there have been arguments that the spin current would create the electromotive force by a modified Ampère's law based on the assumption that the spin current is equivalent to the flow of two monopole charges [46]. Quantum mechanics tells us that this idea is too naive, since the spin is a point object and thus the separation between the magnetic charges are strictly zero, even if one dares to interpret spin by two magnetic charges. The monopole current associated with the spin current is therefore absolutely zero. In fact, spin current is a quantity in an SU(2) spin space which couples to the electromagnetism only when properly projected, and since we know from the argument by Volovik [9] that the projection in the adiabatic limit gives rise only to a hedgehog monopole current but not the spin current. Our study has proved that the correct projection of the magnetic systems with the spin-orbit interaction introduces a magnetic monopole in electromagnetism.

VIII. SUMMARY

As we have discussed, magnetic monopoles are common objects in solids; they emerge in ferromagnetic metals from topological spin structures and from dynamic spin (magnetization) with damping. Although monopoles have been discussed so far based on a gauge theory, we have presented a novel method to access monopoles based on a transport calculation.

In monopoles in ferromagnets, of particular interest are spin damping monopoles, generated in a simple system of a ferromagnet and heavy metal. They open a novel path for connecting magnetism and electronics via an analog of Ampère's law. A novel concept of monopolotronics, control of monopoles, proposed in Ref. [10], is expected to be useful for realizing next generation spintronic devices.

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Appendix A: Electromagnetic fields in the relativistic representation

In this section, we briefly summarize the electromagnetism theory in the relativistic representation using 4-vectors. We follow the convention used in Ref. [7]. Contravariant vectors, such as $x^\mu = (t, x, y, z)$, are represented by the upper indices. Covariant vectors, represented by lower indices, are defined by multiplying the metric tensor $g_{\mu\nu}$ as, for example, $x_\mu = g_{\mu\nu}x^\nu$, where

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (\text{A1})$$

Thus covariant vector of the 4-dimensional coordinate is thus $x_\mu = (t, -x, -y, -z)$. The product of the covariant vector and contravariant vector is a Lorentz invariant scalar, e.g., $x_\mu x^\mu = t^2 - \mathbf{r}^2$. (Note that we use relativistic notation only in this section. In other sections, upper and lower indices of the three-dimensional vectors means the same.) Differential operators are

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right), \partial^\mu \equiv \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z} \right). \quad (\text{A2})$$

We first describe the electromagnetism without monopole, namely, when there is. The electromagnetic field tensor is defined by a U(1) gauge field A^μ as $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Its components are (noting $\partial_i = -\partial^i$) $F^{ij} = -\epsilon_{ijk} B^k = F_{ij}$, where $B^k = (\nabla \times \mathbf{A})^k$, and $F^{0i} = (\partial_t \mathbf{A} + \nabla \phi)^i = -E^i = -F_{0i}$. In the matrix representation,

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}. \quad (\text{A3})$$

The field strength tensor satisfies by definition the following identity

$$\partial_\mu \tilde{F}^{\mu\nu} = 0, \quad (\text{A4})$$

where ($\epsilon^{0123} = 1 = -\epsilon_{0123}$ is the four-dimensional antisymmetric tensor)

$$\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}. \quad (\text{A5})$$

The 0-component of the identity (A4) is

$$\partial_i \tilde{F}^{i0} = -\frac{1}{2} \epsilon^{ijk} \nabla_i F^{jk} = \nabla \cdot \mathbf{B} = 0, \quad (\text{A6})$$

and i -component reads

$$\begin{aligned} \partial_0 \tilde{F}^{0i} + \partial_j \tilde{F}^{ji} &= \frac{1}{2} (\partial_0 \epsilon^{ijk} F_{jk} - 2\partial_j \epsilon^{ijk} F_{0k}) \\ &= -\frac{\partial}{\partial t} \mathbf{B} - \nabla \times \mathbf{E} = 0. \end{aligned} \quad (\text{A7})$$

Therefore, the condition of no monopole, $\nabla \cdot \mathbf{B} = 0$, and the Faraday's law are trivial result of the U(1) gauge symmetry (definition of $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$).

Appendix B: Spin damping

Let us look into a role of spin damping represented by a vector $\mathbf{N} = \mathbf{S} \times \dot{\mathbf{S}}$. A spin dynamics is induced by magnetic field. An equation of motion for a spin is thus generally given by Landau-Lifshitz equation

$$\dot{\mathbf{S}} = \gamma \mathbf{B}_{\text{tot}} \times \mathbf{S}, \quad (\text{B1})$$

where γ is gyromagnetic ratio and \mathbf{B}_{tot} represents the total magnetic field acting on \mathbf{S} . \mathbf{B}_{tot} thus includes an external magnetic field, an internal field due to exchange interaction with other localized spins, magnetic anisotropy field, and also the effect of coupling to other degrees of freedom such as conduction electrons and phonons.

We know that damping or dissipation arises in general from a coupling to an environment, namely, other degrees of freedom. In a case of spins in metallic magnets, most strong source of dissipation is conduction electron. Dissipation in this case is caused by spin-orbit interaction, which converts spin angular momentum into orbital angular momentum. Therefore, damping effect is calculated by evaluating an effective magnetic field from conduction electrons, resulting in [47]

$$\gamma \mathbf{B}_{\text{el}} = -\alpha \dot{\mathbf{S}}, \quad (\text{B2})$$

where α is a constant proportional to the spin-orbit interaction. The damping torque is thus represented by $\alpha \mathbf{S} \times \dot{\mathbf{S}}$, called the Gilbert damping term. The equation of motion thus reads

$$\dot{\mathbf{S}} = \gamma \mathbf{B} \times \mathbf{S} + \alpha \mathbf{S} \times \dot{\mathbf{S}}, \quad (\text{B3})$$

where $\mathbf{B} \equiv \mathbf{B}_{\text{tot}} - \mathbf{B}_{\text{env}}$ is the field neglecting the effect of the environment, \mathbf{B}_{env} . Equation (B3) is called Landau-Lifshitz-Gilbert equation.

The effect of Gilbert damping torque is understood by looking into Fig. 12. In fact, a vector $\mathbf{S} \times \dot{\mathbf{S}} \equiv \mathbf{N}$ tends to point the spin perpendicular to its precession direction, i.e., to the equilibrium direction along the field \mathbf{B} , and hence $\alpha \mathbf{N}$ represents the spin dissipated by the environment.

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